Virtual SEA for noise prediction and structure borne sound modeling

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Introduction
In the mid-frequency range (200-2000 Hz), the difficulties encountered when modeling the car body structural interactions as well as the related acoustic radiation properties are mainly due to the complexity of automotive structures. Previous work [1, 2] has proven the predictive efficiency of an energy-based representation of these interactions within the SEA framework (Statistical Energy Analysis). Nevertheless, classical SEA modeling [3] is submitted to some drawback. Before any SEA modeling, it is required to separate the system domain into mechanically weakly coupled regions. It is only at this stage, that a region can be considered as a SEA subsystem. This first modeling step is critical for several reasons:
• There is presently no reliable theory that mathematically defines the notion of “weakly coupled systems”.
• The illicit partition a single-piece homogeneous subsystem -such as a beam, a plate or an acoustic cavity- into separate SEA subsystems may lead to some physical inconsistency.
• The gathering of subsystems may lead to errors in the vibration level prediction.
Moreover, the practice shows that the discontinuities able to generate an energy gap (condition for the existence of a power flow between subsystems) are more numerous at high frequencies. Consequently, the number of subsystems should increase with frequency.
In the case of a complex structure such as a car body, a well-suited system decomposition is especially difficult to find, due to the variety of structural components and assembly details. It is only up to 2000-4000 Hz that a car body degenerates into a set of weakly-coupled homogeneous plates or shells, easy to identify as separate regions. At lower frequencies, creating relevant SEA models -independent of user's choice-, is a real challenge.
Sub-structuring is not the only difficulty when modeling an automotive structure with SEA: the isolated subsystems often differ from “known” SEA kind of subsystems. Classical SEA components modeling rely on wave-based models; such models only apply for regular topologies when a car body is polymorphic.
Virtual SEA (VSEA) was introduced in order to overcome these limitations by using a Finite Element Method to describe the dynamical behavior of any structure [1, 2]. Then, the proposed method applies to any structural problem that can be addressed using a FE modeling. VSEA provides an automatic sub-structuring of the studied structure, as well as SEA parameters of subsystems, adjusted to the FE model dynamical behavior. VSEA is shortly described in the first part of the paper.
Nevertheless, up to this stage, VSEA was restricted to structural vibrations of the system associated to the FE model. No sound radiation -i.e. coupling with a cavity- nor system extension –adding new subsystems- was possible. The modeling of insulating treatments, that slightly interact with the structure but change the acoustic radiation, could not be achieved too, since the only practicable modeling at mid-high frequencies relies on a wave modeling of the structural vibrations [4, 5]. To above features are mandatory when one wish to get involved into
a vehicle design process. Therefore, the coupling of VSEA subsystems with wave-based analytical models has been investigated and is presented in the second part of the paper.

1. Virtual SEA modeling

To overcome the previously denoted difficulties and limitations of SEA, while staying within the expertise domain of automotive NVH engineers, a specific predictive tool has been developed to translate the dynamic information contained in a Finite Element model into a SEA model. This technique, called Virtual SEA, makes the information provided by the FE model readable and, coupled with a SEA modeler, provides a powerful simplified analysis tool. Any FE model, whatever its complexity, can thus be processed thanks to an automatic sub-structuring algorithm and a built-in modal synthesis solver. The obtained SEA model is robust, and doesn’t require specific skills.

1.1. Experimental SEA reminder

VSEA was initially derived from Experimental SEA (ESEA) [6]: once given the sub-structuring, the average squared transfer matrix between subsystems, \( T^2 \), is built from the point to point FRF matrix \( H \), in each frequency band \( B \), as:

\[
T^2 = S_H H^2 S_H^T
\]

where \( S_H \) is the sub-structuring matrix that gathered observation points \( M \):

\[
S_{H,i} = \begin{cases} 
1 & \text{when } M_k \in \Omega_i \\
0 & \text{when } M_k \notin \Omega_i 
\end{cases}
\]

Subsystems \( \Omega_i \) are such as \( \Omega = \bigcup \Omega_i \), and \( N_i \) is the number of observation points in the subsystem \( \Omega_i \).

The Energy transfer matrix \( E \) is obtained by introducing the equivalent masses of subsystem, as a diagonal matrix \( m \):

\[
E = mT^2
\]

The power input matrix is the averaged input mobility diagonal matrix, \( Y \), provided the measured FRF’s are the responses to unit forces:

\[
\Pi = YI
\]

The loss matrix \( L \) characterizes the SEA model that may be written

\[
\Pi = \omega_B LE
\]

where \( \omega_B \) is the central angular frequency of the frequency band \( B \) (1/3rd octave bands are generally considered), \( \omega_B = 2\pi f_B \). Later, the index, \( B \), will be omitted.

The loss matrix is build from loss factors (damping loss factor, \( \eta_i \); coupling loss factor, \( \eta_{ij} \)) as follows:
Coupling loss factors are assumed to satisfy the well-known SEA reciprocity relationship:

\[ n_i \eta_{ij} = n_j \eta_{ji} \]  \hfill (6)

when introducing the modal density of subsystem \( i, n_i \).

The Loss matrix (SEA model) can be identified from the measured input mobilities and transfer functions by solving an optimization problem:

\[ \hat{\eta}_{ij} = \text{ArgMin}_{\eta_0} \left( \left\| L - \frac{1}{\omega_B} \mathbf{E}^{-1} \mathbf{Y} \right\| \right) \]  \hfill (7)

The loss matrix can not be obtained from a simple matrix inversion, since measurement errors are unavoidable and since the physical relevance of an SEA problem is not granted. A specific procedure involving a Monte-Carlo simulation is used. It relies on a procedure proposed by Lalor [6].

The main practical difficulty of ESEA is to obtain a consistent set of input data, meaning the sum of dissipated power is equal to the input power. Non-consistent data generally leads to non-physical results such as negative loss factors.

Of course, the ability of a given partition to fit SEA theory is another cause of discrepancy in the model identification.

1.2. Virtual SEA introduction

In its early version, VSEA [1] tackle ESEA main uncertainties:

- the transfer matrices are deduced from a conservative FE model, so that the associated damping is known
- an automatic, optimal, sub-structuring procedure is provided

Nevertheless, when dealing with highly non-homogeneous systems, one observes large spread in vibration levels within the subsystems. This may lead to mean quantities estimation errors when sampling the vibratory field. This heterogeneity of the car body structural behavior –and response- has been specifically addressed by improvements of the Virtual SEA method [2].

The main feature of the improved version of VSEA is to consider the so-called modal energies as main variables. Local modal energies are related to the point-to-point response as:

\[ e_n = \frac{1}{4} \mathbf{Y}^{-1} \mathbf{H}^2 \mathbf{Y}^{-1} \pi \]  \hfill (8)

After averaging over subsystems, one gets:

\[ \overline{e_n} = \frac{1}{4} S_\pi \mathbf{Y}^{-1} \mathbf{H}^2 \mathbf{Y}^{-1} S_\pi \overline{\pi} \]  \hfill (9)

Using the modal energy as main variable, the SEA model may be set in the form:
\[
\overline{\Pi} = \omega_{B} L_{n} \overline{E}_{n}
\]  

(10)

where \( L_{n} = \text{diag} \{ n \} L \) is the symmetrical modal coupling loss matrix, with \( n \) the vector of subsystems modal density.

The identification of the computed expression (9) to the SEA model defined by expression (10) leads to the new optimization problem to be solved:

\[
\hat{L}_{n} = \text{ArgMin}_{L_{n}} \left( \left\| L_{n} - \frac{1}{4\omega_{B}^{2}} [S_{i} Y^{-1} H^{2} Y^{-1} S_{i}] \right\| \right)
\]

(11)

Using this new procedure, it appears that the variance within each subsystem is decreased, as shown on figure 1, providing a safer inversion procedure. The global dynamics is decreased while homogeneous areas seem to rise.

Figure 1: On the left transfer mobility matrix of a car body \( H^{2} \) averaged over 630 Hz 1/3 octave band. On the right, transfer modal energy matrix \( Y^{-1} H^{2} Y^{-1} \)

Figure 2: On the right, matrix error between the initial (FE computed) modal transfer energy matrix and the reconstructed modal transfer energy matrix (SEA model). Color scale is in dB. On the left, percentage of reconstructed transfer within a given error interval in dB
Thus results quality improves as shown on figure 2; the identification procedure now leads to an error smaller than 1 dB for 95% of the subsystems modal energy transfer, in the case of a car body in white.

Moreover, with VSEA it now becomes possible to reconstruct local point vibrations from the subsystems averaged modal density, when inverting expressions (9) and (10):

$$\hat{H}^2 = Y S_{\Omega}^{++} L_{\Omega}^{--} S_{\Omega}^{++} \Omega\ Y$$

where $S_{\Omega}^{++} = (S_{\Omega}^t S_{\Omega})^{-1} S_{\Omega}$ is the pseudo-inverse of $S_{\Omega}$

It appears that the pseudo-inverse may be computed easily, and it is defined by:

$$S_{\Omega,k}^{++} = \begin{cases} 0 & \text{when } M_k \notin \Omega_i \\ 1 & \text{when } M_k \in \Omega_i \end{cases}$$

VSEA can thus be seen as a data reduction: the initial squared transfer matrix, $H^2$, can be reconstructed, $\hat{H}^2$, using only the SEA model (statistical estimation of modal energy transfer between subsystems) and local input mobilities. In the studied case of a car body, the initial information consisted in about 1200x1200 spectra; the VSEA model only requires a 34x34 sparse matrix plus 1200 input mobility to provide an estimate of the initial information. The information reduction factor is about 1000.

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Figure 3: Virtual SEA model of a Peugeot 207 body-in-white. Observation point automatic grouping. Left: example of identified subsystems (Blue: windscreen. Green: roof. Red: left side frame); Right: model view as a 3D network.

Moreover, the SEA theory ensures subsystem characterization is intrinsic; this implies that a modification of one subsystem - during a design process- will only affect its internal SEA characteristics as well as its coupling factors with connected subsystems.

The reduced variance of the processed data also improves the performance of the automatic subsystem partition procedure. As far as the automatic sub-structuring is part of the systems...
identification process described by expression (11), a decreased of the input data variance leads to a more robust model. Until now, no specific sub-structuring performance indicator has been developed: both sub-structuring and SEA model identification are evaluated at the same time in the reconstruction error presented on figure 2. 95% of energy transfer between subsystems is modeled within 1 dB, showing a good agreement between the proposed model and the SEA theory.

Figure 3 (left) illustrates the observation nodes grouping, determining the subsystems extend. Figure 3 shows the 3 subsystems -namely the roof, the windscreen and the left side frame (B-Pillar and side rail) that will be investigated later in terms of acoustic radiation.

2. Coupling of a VSEA Model with analytical models

As a consequence of the diffuse field assumption –or high modal density-, most of SEA analytical coupling models are built by using a wave approach of structural vibration fields or acoustic fields. Thus, it is proposed to apply a second modeling step to the VSEA subsystem so that it can be coupled to a wave-based SEA model. To turn a VSEA subsystem model into a wave-based model, one has to compute its dispersion curve $\omega \rightarrow k(\omega)$, where $k$ is the free averaged wave number. Once the wave number is known, it becomes possible to simulate a diffuse field that can be coupled to various kind of subsystems such as beam, plate or cavity. In this paper, only radiation aspects (coupling to cavities) -that are the most relevant to noise prediction problems- will be addressed.

2.1. Relationship between wave numbers and modal densities

In academic vibroacoustic approaches, the wave number is derived from the characteristic equation of the subsystem –either beams, plates, shells, cavities- [7]. The modal density –required for SEA modeling- is derived from wave-numbers using topological and geometrical information. For plates (resp. beams), straightforward expressions are obtained from an asymptotic calculation in the case of a simply supported rectangular plate (resp rectilinear beam):

For flat isotropic plates, the wave number is related to the modal density by the relation:

$$k_{ss} = \frac{2\omega N}{\sqrt{S}} \quad (14)$$

where $S$ is the plate surface.

For rectilinear beams, the wave number is related to the modal density by the relation:

$$k_{ss} = \frac{\omega N}{L} \quad (15)$$

where $L$ is the beam length

These expressions are easy to handle, but they are limited to ideal subsystems geometry. Nevertheless, due to the fact that they are derived from an asymptotic formulation, they are robust to boundary conditions as well as a number of design uncertainties. These expressions can be used extensively as far as the modal density remains high enough within the studied subsystem. Similar expressions can be found in the literature for most of the cases that can be addressed analytically including stiffened plates, shells and some others [8, 9, 10].
Figure 4 compares the wave-number (14) of two plate-type vehicle subsystems (roof and windscreen), to an analytical plate modeling (AutoSEA I). In these cases, modeling plate-type subsystems doesn’t require a high level of expertise, but leads to some approximations as, in facts, the roof and windscreen are shells. For both, VSEA provides a cylindrical shell-type dispersion curve that remains close to the analytical model. When looking carefully to the windscreen wave-number, it appears that VSEA wave number lies between 2 possible modeling of the windshield as an equivalent homogeneous plate (4.5 mm thick) or as a sandwich (glass-PVB-glass). One can clearly observe that the VSEA wave-number asymptotically tends to the sandwich behavior that, in facts- degenerates to single sheet wave number (2mm thick).

Figure 5 compares the wave-number of a shell-type vehicle subsystem (left side frame isolated in the first section) to an analytical shell modeling (AutoSEA I). In this case, the analytical modeling requires a high level of expertise, but also provides a result very close in magnitude to VSEA. The only unexpected fact, here, was that the T-shape structure was only one SEA subsystem.

These figures show that the VSEA modeling –including auto sub-structuring and SEA modeling- accounts for the detailed structural behavior, inherited from the Finite Element model, without any specific SEA knowledge. The additional information is limited to the typology of the subsystem, and geometrical descriptors (length, area…), requiring a limited expertise level. These results are not a true validation of the proposed method, since the concepts discussed here (modal density, wave-numbers) can not be measured. Provided analytical models are known to be robust, the observation of the same trends and order of magnitude is encouraging.

![Wave number (m-1)](image)

**Figure 4**: Wave numbers of the roof and the windshield deduced from an analytical flexural plate modeling compared to virtual SEA results
2.2. Coupling a VSEA model to a cavity

Sound radiation is a key-point in any vibroacoustic computation as far as noise prediction is generally the goal of vibroacoustic studies. Until now, VSEA has only been applied to structural systems, although it could certainly apply in the case of FE problems including acoustic cavities. Nevertheless, this would be of little interest in an NVH design process. Indeed, NVH design mainly concerns acoustic trim material that is laid between the structure and the cavity. Due to the variety of layered insulating materials that can be designed, the use of the infinite multi-layered media modeling [4, 5] is considered as a good trade-off between model simplicity and accuracy. Moreover, recent work by Villot and all [11] greatly improved the quality of the modeling by including effects of the size of the radiating panels. This work is the main reference of the derivations below. In this paper, only the case of a bare structure is considered, but the theory is supporting multi-layered insulating structures (work is in progress). Additional corrections are also introduced to improve the accuracy of the fluid-structure interaction prediction in the medium frequency.

Finite-sized window convolution effect is generalized to all coupling loss factors predicted by the wave theory in the (infinite) Space Fourier Transform's domain.

This section is dedicated to the optimal use of VSEA information to model fluid-structure interaction.
2.2.1. **SEA fluid-structure coupling basics**

First, it is stated, as far as acoustic radiation is a surface interaction, that only radiation of surface structural subsystems will be considered. Using SEA, the acoustic radiation of a vibrating structure is characterized by an acoustic power, $\pi_{rad}$. The radiated power is related to the structure energy by the coupling loss factor, $\eta_{rad}$, such as:

$$\pi_{rad} = \eta_{rad} \omega \varepsilon$$

(16)

where $\varepsilon = m \bar{v}^2$ is the structure total energy; $m$ is the mass of the structure and $\bar{v}^2$ is the mean quadratic velocity of the structure.

The radiation efficiency, $\sigma$, is a non-dimensional number often used to assess radiation properties of vibrating structures. It is defined as the ratio to the actual radiated power to the ideal power radiated by an equivalent 1-D radiation problem (rigid piston with the same area and the same mean quadratic velocity radiating in a tube). By definition [12],

$$\sigma = \frac{\pi_{rad}}{\rho c S \bar{v}^2}$$

(17)

Where $\rho$ is the fluid specific mass, $c$, its sound celerity, and $S$ is the structure radiating area.

The radiation coupling loss factor may then be calculated form relations (16) and (17).

$$\eta_{rad} = \frac{\sigma \rho c S}{\omega m}$$

(18)

Analytical expressions of the radiation efficiency may be found in the acoustic literature for different kind of canonical resonant structures (plate, shell...): the most famous ones are due to Maidanik [9], other formulas may be found in [12]. The radiation efficiency tends to one at frequencies above a critical frequency.

The reversed problem of the excitation of a structure by a diffuse acoustic field is solved by considering the reciprocity principle (6).

2.2.2. **Statistical radiation efficiency computation**

Classically, it is assumed that the sound radiation is well enough determined when considering a wave approach (k-space) of the fluid-structure coupling. Thus, the diffuse (SEA) vibration field of a flat plate is modeled as a sum of uncorrelated waves, freely propagating in any direction and characterized by the wave number $k(\omega)$. The classical formulations are greatly improved when considering a spatial windowing effect [11], namely finite-sized panels. Assuming the vibration field of a flat structure can be described as a diffuse -isotropic -distribution of freely propagating waves, the radiation efficiency over a rectangular window appears to be [11]:
Expression (19) underlying assumption is closely related to SEA, since it refers to a statistical description of a resonant vibrations field that may be characterized either by the mean behavior of randomly distributed waves or the mean response of random modes. This is why expression (19) is the most appropriate, given a SEA description of structural vibrations. Moreover, the formulation that is used to derive expression (19) may be extended to multi-layered structures.

It is finally recalled that the wave-number, \( k(\omega) \), may be calculated from the modal density, itself computed from the FE model, requiring no specific skills for SEA model parameters setting.

Figure 6: Statistical radiation efficiency of the roof. Left: Analytical computation of the radiation efficiency over structural wave incidence angle \( \psi \); Blue, mean value, dotted red, upper and lower standard deviation limits. Right: radiation efficiency vs. incidence in two frequency bands (8 kHz and 25 kHz).
Figure 7: Statistical radiation efficiency of the windshield. Blue, mean value, dotted red, upper and lower standard deviation limits.

Figure 6 and 7 show examples of statistical radiation efficiency, computed using expression (19) for the roof and the windshield previously isolated. Standard deviation is obtained from the average over 50 $\psi$ angles (incidences of the propagation of the structural wave within the plate plane) and over 10 internal frequency within the related 1/3rd octave bands. For the roof, the high standard deviation is due to a high peaks of radiation (directivity) below the critical frequency as shown on Figure 6 (Right). Above the critical frequency, the directivity is broadband.

Related coupling loss factors are compared to the same quantities computed by the well validated analytical software AutoSEA1 using (18) in figure 8. Results are very similar in the considered cases and when some difference is observed (as in between roof CLF’s) it falls within the standard deviation as given in Figure 6.
2.2.3. Advanced radiation efficiency computation

In the previous paragraph, the statistical radiation efficiency was introduced. It is the most probable value considering the available information: the wave number deduced from the modal density and an isotropic diffuse field assumption. Nevertheless, this assumption may be far from reality; for example, in case of non-isotropic structures or strongly heterogeneous structures, where the radiation efficiency may be related to local effects (edge radiation). In such cases, where the vibration field is sensitive to boundary conditions or local heterogeneities, a wave approach is not valid anymore. A modal approach is then substituted to the previous wave approach; it leads to a mode-per-mode radiation efficiency calculation. In the general case, the modal radiation computation requires a time-consuming Boundary Element modeling, provided the substructure is isolated; this requires setting convenient boundary conditions for the substructure itself but also for the fluid domain. Such an approach will not be developed here. We will consider the simplified case of plane baffled structures, which radiation can be computed using a 2-dimensional Fourier Transform. In this case, the radiation efficiency may be written as:

\[
\sigma_i(\omega) = \frac{\omega}{\pi^2 c} \int_{k_x^2 + k_y^2 < \frac{\omega^2}{c^2}} \frac{\left| \psi_i(k_x, k_y) \right|^2}{\sqrt{\omega^2 - k_x^2 - k_y^2}} dk_x dk_y
\]

(20)
where \( \psi_i \) is the Fourier’s transform of a mode shape.

The modal frequencies and mode shapes can be obtained by analytical modeling or be derived from the virtual wave number by assuming modal quantification (following plate typology) of the virtual wave number. The prediction of the radiated power is finally obtained by (21)

\[
\pi_{\text{rad}} = \rho c S \bar{v}^2 \sum_{i=1}^{n} \sigma_i(\omega)
\]

where \( n \) is the number of modes in the studied frequency band and \( \bar{v} \) the space-averaged velocity in the subsystem.

Figure 9 compares the average modal radiation CLF to the statistical CLF for the windshield, modeled, below 1000 Hz, as a rectangular simply supported plate. One observes a good agreement between modal and statistical computation. Differences between statistical and modal predictions give, at least, an indication of the uncertainty due to boundary conditions in radiation computations.

Figure 9: Radiation coupling loss factors of the windshield computed using Eq.(19) (thin magenta), predicted by AutoSEA1 (dotted blue) and using a modal formulation (thick green)
Conclusion

This paper wished to introduce the coupling of Virtual SEA models to cavities. This new feature was the missing piece that opens the field of vibroacoustic modeling using hybrid analytical/FE models. Analytical models are the most wanted in advanced design, because they directly depend on the first order design parameters. They also do not require pre-existent drawings.

The virtual SEA method was first briefly presented. It is born from the association of Finite Elements Method and Experimental SEA. In its last evolution, a point-to-point reconstruction of energy transfer functions is made possible, using an appropriate formulation. Coupling Loss Factors are computed as well as modal densities. The new procedure also improves the automatic sub-structuring process as the variance within subsystems appears to be decreased.

Then, in a second modeling step, a wave-number is derived from the modal density, assuming an isotropic diffuse field. Then, the sound radiation of a vibrating surface through a spatial rectangular window may be computed from an integral expression. Results are shown for typical car-body subsystems that may be modeled analytically. Wave-numbers provided by the VSEA seem realistic compared to analytical ones. Statistical radiation Coupling Loss Factors are then computed from the wave-number dispersion curve. Again results are rather similar to analytical expressions.

Such a wave modeling has been chosen because it accounts for finite size radiation effects and because it allows to consider the introduction of a trim layers.

Then, the radiation CLF may be computed from the modal response of a given flat substructure. Nevertheless, this requires the substructure mesh to be isolated and bounded, with some assumptions.

From the theoretical point of view, two main directions are to be privileged for Virtual SEA development:

- **the hybridizing of VSEA subsystems with analytical subsystems.** It will allow advanced vibroacoustic design, using analytical or parametric SEA subsystem modeling, for parts of a car body to be designed. Trim layers also have to be modeled with high priority since they are a major design parameter.
- **the extensive use of the finite element model** to provide useful data at an acceptable cost. Radiation properties, damping, trim modeling could be reconsidered in a statistical sense, in order to compute, at each modeling step, the most probable result according to SEA.

In a general effort to increase SEA model robustness, further development is currently also performed on the statistical description of energy exchange within SEA subsystems by managing directivity information all along the SEA subsystem chain (such as power emission).

The presented work is still in progress and a full-scale experimental validation of virtual SEA is on the way.
References


[6] Lalor N. Experimental statistical energy analysis: a tool for the reduction of machinery noise, Presented to the E.J. Richards Memorial Session at the 131st ASA Meeting, Indianapolis, USA. May 1996


